ENGR 1181  |  Lab 6: Beam Bending

- Preparation Material
- Lab Procedure
- Report Guidelines
Preparation Material
Learning Objectives:
The Beam Bending Lab introduces basic concepts that engineers use to design structures, including material choices and design configurations. Students will review this document, watch a dial caliper video, and then take the Carmen quiz before arriving at lab.

1. Overview of the Beam Bending Lab
In the Beam Bending Lab, you will:

1. Investigate and apply the concepts of stress, strain, and Young’s modulus for structural materials.
2. Apply the stress-strain equation to calculate how applied forces deform structures.
3. Calculate the moment of inertia of various beam geometries and determine how beam geometry affects the stiffness and strength of beams.
4. Apply forces to cantilever beams and make accurate measurements of the dimensions and deflections of structural beams.
5. Identify unknown beam material through analysis and calculation of Young’s Modulus.
6. Make observations and evaluate behavior of various beams based on material and shape.

2. Engineering Structures
This lab explores the engineering applications of materials and structures. Engineers choose suitable materials – metal, plastic, glass, concrete, etc. – to design useful, economical and safe structures. These structures range from very small – nanotechnology devices and computer chips – to very large – airplanes, bridges and tall buildings. Engineers have to be certain that the structures they design will be stable and safe under the most adverse operating conditions. But first, let’s investigate how structural materials behave when they are pushed and pulled by forces.

Force and Deflection for a Simple Spring
You learned in physics that an applied force $F$ makes a spring stretch (or compress) by the distance $x$. The linear relationship between $F$ and $x$ is Hooke’s Law:

$$F = k \cdot x \tag{1}$$

where $k$ is called the spring constant. Note that for very large values of the spring constant, the spring stretches only a very small amount in response to an applied force.

In SI units, the force $F$ is in Newtons, distance $x$ is in meters, and the spring constant has units of N/m.

In English units, $F$ is in lbf (pounds-force), $x$ is in inches, and the spring constant has units of lbf/in.
Simplified Model of Structural Materials

It turns out that solid materials also exhibit an elastic, spring-like response to applied forces. Figure 1a (below) shows an aluminum rod with cross-section area $A$ and initial length $L$ when no force is applied. The atoms in the rod are all connected to their nearest neighbors by electromagnetic forces that are "spring-like" – here shown as atoms connected by springs.

Figure 1a. Aluminum Bar, no Force

Figure 1b. Aluminum Bar, Force $F$

Figure 1: Aluminum Bar with and without an Applied Force

Figure 1b shows the same aluminum rod, but with two opposing forces $F$ applied at each end. In response to the forces, the length of the aluminum bar *increases* (stretches) by an amount $\Delta L$. When the forces *pull outward*, as shown in Figure 1b, the bar is said to be in "tension".

If both forces were reversed to *push inward* on the ends of the bar, the length of the bar *decreases* by the same amount $\Delta L$ and we would say that the bar is in "compression".

Stress, Strain and Young’s Modulus

For Figure 1, the "spring-like equation" for the aluminum bar is

$$\frac{F}{A} = E \frac{\Delta L}{L}$$

(2)

The proportionality constant $E$ is called *Young’s modulus*, or the *modulus of elasticity*, and has units of $N/m^2$ (Pascals) or $lbf/in^2$ (pounds per square inch, or simply psi).

The ratio $F/A$ is called "stress" and is given the Greek letter symbol $\sigma$:

$$\sigma = \frac{F}{A} \quad \left\{ \frac{N}{m^2} \text{ (Pascals)} \text{ or } \frac{lb}{in^2} \text{ (psi)} \right\}$$

(3)

The physical interpretation of stress is: the *internal pressure* in the aluminum bar caused by *externally applied forces*. 

The ratio \( \Delta L/L \) is called "strain" (note that it has no units) and is given the Greek letter symbol \( \varepsilon \):

\[
\varepsilon = \frac{\Delta L}{L} \quad \left\{ \frac{m}{m} \text{ or } \frac{in}{in} \text{ (dimensionless)} \right\}
\]  

(4)

Therefore, strain is the fractional elongation (or compression) of a structural member caused by applied forces. For instance, a strain of \( \varepsilon = 0.025 \) would correspond to a 2.5% increase in the length of the bar.

Substituting Equations (3) and (4) into (2) gives the fundamental "stress-strain" equation for materials:

\[
\sigma = E \cdot \varepsilon \quad \left\{ \frac{N}{m^2} \text{ (Pascals)} \text{ or } \frac{lbf}{in^2} \text{ (psi)} \right\}
\]  

(5)

For most structural materials (steel, etc.), Young's modulus is very large, so large forces cause only small deformations. Have you ever been in a car on a bridge and felt the slight up-and-down motion of the bridge caused by heavy trucks moving across the bridge?

Metric and English values of Young's modulus for the materials we will use in the Beam Bending Lab are shown in Table 1. Notice that steel is almost three times stronger than aluminum.

<table>
<thead>
<tr>
<th>Material</th>
<th>Young's modulus, ( E )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>GPa ((10^9 \text{ N/m}^2))</td>
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<tr>
<td>Aluminum</td>
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<tr>
<td>Steel</td>
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<tr>
<td>Polystyrene</td>
<td>3</td>
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<tr>
<td>Basswood</td>
<td>35</td>
</tr>
</tbody>
</table>
Stress-Strain Graphs and Structural Failure

Figure 2 shows the graph of stress versus strain for a typical structural material. The slope of the linear portion of the curve is equal to Young's modulus. The linear portion of the curve (up to point 2) is called the *elastic region*, because every time the force is removed, the beam returns to its original length. Point 2 is called the Proportional Limit.

![Figure 2: Graph of Stress vs. Strain](image)

After Point 2, however, the curve becomes non-linear and the beam will be permanently deformed. The graph also shows that if the beam is stressed to Point 4 and the force is removed, then the beam retains a permanent 0.2% elongation. If the beam is stretched to the end of the curve, the beam fails completely and breaks apart.

Engineers design structures so that the maximum stress in buildings or airplanes will never exceed the Proportional Limit (Point 2), even under the most adverse conditions. Professional design standards, building codes and governmental regulations also require structural engineers to use safety factors in the design of bridges, buildings, airplanes and other critical structures.
Sample Calculation of Stress and Strain

Let’s use the stress-strain equation to solve a sample problem.

Consider a round aluminum bar (shown in Figure 3) that has an initial length $L = 1 \text{ m}$ and a cross-sectional area $A = 10 \text{ mm}^2$. Its upper end is fixed to the ceiling and the weight $(m = 100 \text{ kg})$ hangs on the free end, as shown in Figure 3.

**Calculate:** (1) the stress $\sigma$ in the bar, (2) the strain $\varepsilon$, and (3) the elongation $\Delta L$.

(1) **Stress in the bar**  
The force of gravity on the mass is  
\[
F_g = m \cdot g = (100 \text{ kg}) \cdot (9.8 \text{ m/s}^2) = 980 \text{ N}
\]

\[
\text{Stress} \quad \sigma = \frac{F_g}{A} = \frac{980 \text{ N}}{10^{-5} \text{ m}^2} = 9.8 \times 10^7 \text{ Pascals}
\]

This force pulls down on the end of the bar. The upward, opposing force is provided by the ceiling. Therefore, the internal Stress in the bar $(A = 10 \text{ mm}^2 = 10^{-5} \text{ m}^2)$ is

\[
\text{Strain} \quad \varepsilon = \frac{\sigma}{E} = \frac{9.8 \times 10^7 \text{ Pa}}{70 \times 10^9 \text{ Pa}} = 0.0014 = 0.14\%
\]

(2) **Strain in the bar**  
Since Young’s modulus for aluminum is $E = 70 \text{ GPa}$

(3) **Elongation**  
The strain $\varepsilon$ is equal to the fractional elongation, $\varepsilon = \Delta L/L = 0.0014$. And since we know the initial length of the bar, $L = 1 \text{ m}$, then the elongation $\Delta L$ is

\[
\text{Elongation} \quad \Delta L = \varepsilon \cdot L = (0.0014) \cdot (1.0 \text{ m}) = 0.0014 \text{ m} = 1.4 \text{ mm}
\]

**Student Exercise**

Using the method from the example above, find stress, strain and elongation for a suspended aluminum bar $(L = 5 \text{ ft}, A = 0.01 \text{ in}^2)$ that holds a weight of mass $m = 100 \text{ lbm}$. Do your calculations in English units. Note that the force of gravity on a $100 \text{ lbm}$ mass is $F_g = 100 \text{ lbf}$.

Check the correct answer, $\Delta L = (A) 0.006 \text{ m} \quad (B) 0.060 \text{ ft} \quad (C) 0.060 \text{ in} \quad (D) 1.5 \text{ mm}$
3. Cantilever Beams

Horizontal Cantilever Beam

A cantilever beam is a structure with one end firmly anchored and the other end free to move. Figure 4 shows a cantilever beam with the beam oriented in a horizontal plane.

The free end of the beam will move down if an external force $F$ is applied to the end. The deflection of the free end of the beam due to the applied force $F$ depends on: (1) the dimensions of the beam (length $L$, width $w$, and thickness $t$), (2) Young’s modulus ($E$) for the beam material, and (3) a geometry factor called the Moment of Inertia.

Examples of horizontal cantilevers are: airplane wings, diving boards, and the overhanging section of the upper level deck in Ohio Stadium.

Figure 4: Horizontal Cantilever Beam

The force $F$ causes the end of the beam to deflect downward by an amount $\delta$. The equation to calculate the deflection $\delta$ is:

$$\delta = \frac{FL^3}{3EI} \quad \{ \text{m or in} \}$$  \hspace{1cm} (6)

The Moment of Inertia, $I$, is a geometry factor that depends only on the cross-sectional dimensions (width $w$ and thickness $t$) of the beam. For the rectangular beam shown in Figure 4, the moment of inertia is

$$I = \frac{wt^3}{12} \quad \{ \text{m}^4 \text{ or in}^4 \}$$  \hspace{1cm} (7)
Sample Calculation of Deflection

Given a rectangular aluminum beam with \( L = 1 \text{ m}, \) \( w = 5 \text{ cm}, \) \( t = 1 \text{ cm} \) and \( F = 100 \text{ N}, \) first calculate \( I: \)

\[
I = \frac{wt^3}{12} = \frac{(0.05 \text{ m})(0.01 \text{ m})^3}{12} = 8.33 \times 10^{-8} \text{ m}^4
\]

Then calculate the deflection at the end of the beam (Young’s modulus for aluminum is in Table 1):

\[
\delta = \frac{FL^3}{3EI} = \frac{(100 \text{ N})(1 \text{ m})^3}{3(70 \times 10^9 \text{ N/m}^2)(8.33 \times 10^{-8} \text{ m}^4)} = 0.0057 \text{ m} = 5.7 \text{ mm}
\]

Vertical Cantilever Beam

In the Beam Bending Lab, you will test vertical cantilever beams, similar to the beam in Figure 5. In the figure, the force is horizontal and the beam bends to the right.

A few familiar examples of vertical cantilevers are:

- Trees
- Stop signs
- Tall buildings
- Wind turbine towers

On a windy day, the force of the wind is distributed over exposed surfaces and causes these structures to bend.

The Sears Tower in Chicago (now called the Willis Tower) is 110 stories and 1,450 feet tall. A 60 mph wind causes the building to bend and the top of the tower to move laterally by 6 inches. The tower’s structure is designed to safely withstand the largest wind speed ever expected in Chicago.
4. Lab Preparation
Theoretical Model of the Beam Bending Apparatus

In the beam bending apparatus, the force \( F \) is applied at a location that is a distance \( L = 8.75" \) from the fixed end of the beam, as shown in Figure 6. Also shown, the deflection of the beam is measured (by the Dial Indicator) at the location \( S = 7.5" \) from the fixed end of the beam.

The equation that gives deflection as a function of the applied force and beam properties is:

\[
\delta = \frac{F S^2}{6EI} (3L - S) \quad (8)
\]

Using Equation 7 and then 8, do a sample calculation for a steel beam (as used in this lab) that has the following properties:

- \( L = 8.75" \)
- \( S = 7.5" \)
- \( w = 0.50" \)
- \( t = 0.125" \)
- \( E = 27,000,000 \text{ lbf / in}^2 \)
- \( F = 1.0 \text{ lbf} \)

Moment of Inertia:

\[
I = \frac{wt^3}{12} = \frac{(0.5 \text{ in})(0.125 \text{ in})^3}{12} = 8.138 \times 10^{-5} \text{ in}^4
\]

Then, calculate deflection:

\[
\delta = \frac{F S^2}{6EI} (3L - S)
\]

\[
\frac{(1.0 \text{ lbf})(7.5 \text{ in})^2}{6(27,000,000 \text{ lbf / in}^2)(8.138 \times 10^{-5} \text{ in}^4)(3 \cdot (8.75 \text{ in}) - 7.5 \text{ in})}
\]

\[
\delta = 0.080 \text{ inches}
\]
Moment of Inertia for Various Beam Geometries

In actual practice, a beam with a solid rectangular cross-section is not always the strongest or the most economical for use in structures.

For instance, the vertical support tower for a wind turbine is usually made of steel and has a tapered, hollow, circular cross-section. Because of its circular symmetry, it has the same strength regardless of which way the wind is blowing. A typical wind turbine is shown in Figure 7.

Many beams used in bridges and buildings have the cross-section of the block letter "I" and for that reason are called "I-beams".

For specific applications, beams with many different shapes and geometries are designed to be light, strong and as economical as possible. Figure 8 shows the cross-section geometries of a hollow circular beam, an I-beam and a box-beam.

![Figure 8: Cross-section Geometries of (1) Hollow Circular Beam, (2) I-Beam, and (3) a Box Beam](image)

Lab Preparation Assignment

In order to finish your preparation for the Beam Bending Lab, students are required to:

- Watch the dial caliper video
- Take the Beam Bending Lab quiz (on Carmen)
- Preview the following Beam Bending Lab Procedure
Lab Procedure
Introduction and Background
A large commercial real estate firm bought up multiple properties in the old industrial end of town. This includes some brownfields and abandoned manufacturing plants and warehouses. They believe this area will see a rebirth in the next 5-10 years and see this as a large reclamation project. Mixing old industrial space and future sustainable practices, they want to reuse as much of the material as possible. However, the exact condition of the property is not known, including soil contamination, water infiltration, building structure integrity, and reusability of materials. Your engineering firm was hired to evaluate all aspects of this project and provide guidance in possible re-use of the materials.

Your boss has asked a small group of engineers (including you) to go out and evaluate the buildings for material assessment and possible reusability. Once in the field, you identify some of the structural beams and various raw materials that were never used in the largest building, which was a metal forming plant.

Your task is to first identify this unknown material through gained lab knowledge, and then report back to the client on ways to reuse and incorporate the material in future projects. The client has asked that each engineer in your group provide their own individual conclusions and recommendations.

Photo credit: “Abandoned VI” by Jakub Kubica
ENGR 1181 Lab 6: Beam Bending
Lab Procedure

Lab Setup: Assemble the Beam Bending Lab Kit & Open the Worksheet
Working in a team of two, open the Beam Bending Lab Kit and examine the contents. If your Beam Bending Lab apparatus is not assembled, then assemble the parts to match Figure 9. The following components are included in your kit:

- Clamp – used to securely hold the fixed end of the vertical cantilever beam
- Vertical Cantilever Beam – made of different sizes, shapes, and materials, including:
  - Aluminum
  - Copper (rectangular beam)
  - Copper (closed square beam)
  - Unknown
- Dial Indicator – a precision instrument used to accurately measure beam deflection
- Pulley – transmits the vertical force of the weights to a horizontal force on the beam
- Weight Holder – holds up to ten weights
- Extra Weights – ten weights are supplied with each lab kit (50 gm. each)
- (1) a 5/32” Hex wrench (T-shaped tool)

Let your instructor know immediately if anything is missing from the kit.

Figure 9: Beam Bending Lab Apparatus
ENGR 1181 Lab 6: Beam Bending

Lab Procedure

Open the Excel file called "Beam_Bending_Lab_Worksheet" located on the EEIC website and save it to your Z : Drive or to a USB memory device.

Take minute or two and look at what is in the worksheet and how it is organized. You are going to use the worksheet to record your data and take it with you after you have finished the lab. It's always good practice to save the worksheet often!

Notice that there are places to enter data for each of Tasks 3 through 6. The cells where you are to enter data are marked with a blue background. The rest of the cells contain formulas for calculations after you enter data. Also, the first recorded task (Task 3) has a graph that will automatically plot your data as you enter it. You will eventually create a similar graph for each of the subsequent tasks.

Task 1. Measure the characteristics of the Aluminum Rectangular Beam

1.1 Locate the Aluminum Rectangular Beam. See Figure 10 that describes thickness (t) and width (w). Using the Dial Caliper, measure the t and w of the beam and enter these values into Excel worksheet, Table 1.1, cells F14 and F15.

   Notice that values for distance to force (L), distance to dial indicator (S), and force (F) are already entered. Check to make sure this matches the actual set-up. Also, notice that the spreadsheet automatically calculates the Moment of Inertia (I) for the aluminum beam.

   Figure 10: Beam Measurements

1.2 Loosen the four screws at the left end of the clamp (if necessary), and insert the aluminum beam into the clamp. Make sure the hole in the beam is at the top and the bottom end of the aluminum beam is fully inserted into the clamp.

   Gently tighten the cap screws making them just tight enough to hold the beam securely and vertically in place. Do not make them "gorilla tight".
ENGR 1181 Lab 6: Beam Bending
Lab Procedure

1.3 Before the dial indicator was loaded (no weight added or placed against beam), it should have read approximately -0.010", but will be vary different indicators. Note that the small dial (in tenths of an inch) is just shy of 0 because the large dial (thousandths) is also shy of zero.

**DO NOT ROTATE THE BEZEL ON THE FRONT OF THE DIAL INDICATOR.** The black marker should always be in line with the plunger, as shown in Figure 11.

Once it is placed in the apparatus and the plunger is engaged on the beam, it should move to near zero or a small positive value. It’s okay that it’s not exactly zero because all of the measurements taken are relative and will be calculated based on the change from the initial “zero weight” loading. The reading shows that the plunger is engaged and in contact with a surface, as shown below:

![Unloaded Dial Indicator](image1) ![“Zero Weight” Dial Indicator](image2)

**Figure 11: Dial Indicator: Unloaded and Engaged (or “Zeroed”)**

1.4 Make sure that the measuring rod on the dial indicator touches the *middle* of the beam, in reference to the width (w).

1.5 Feed the "eye-bolt" end of the weight holder through the hole in the beam and thread the thumb nut onto the eye-bolt thread. Also, make sure the string goes over the top of the pulley and the weight holder hangs to the right. This is shown in Figure 9.
Lab Procedure

1.6 Read the dial indicator (with no weight added) and enter this number into Table 1.3, cell E26.

1.7 Add one 50 gm weight to the weight holder and observe that: (1) the beam moves slightly to the right and (2) the dial indicator reading increases.

1.8 Lightly tap on the weights and/or tap on the base of the beam bending apparatus. Notice that the reading on the indicator changes slightly (will usually increase).

This occurs because of internal friction inside the dial indicator and the pulley, and also internally with the dial indicator mechanism. The technical term for this is called "Measurement Hysteresis". Giving the pulley and apparatus a small vibration allows the system to settle into a more representative reading.

1.9 Measure the deflection (in inches) and record in column E.

1.10 Repeat this measurement process, obtaining readings for all ten weights, adding one weight at a time and recording data at each step.

1.11 Observe the numbers and graph and note any irregularities. Verify that the incremental deflection ($\Delta X$) is relatively constant (linear).

**Task 2. Measure the characteristics of the Copper Rectangular Beam**

2.1 Repeat all of the steps outlined in Task 1 for the Copper Rectangular Beam.

2.2 Create a graph for Task 2, similar to the pre-made graph in Task 1.

2.3 Observe the numbers and graph and note any irregularities. Verify that the incremental deflection ($\Delta X$) is relatively constant (linear).

**Task 3. Measure the characteristics of the Copper Square Beam**

3.1 Repeat all of the steps outlined in Task 1 for the Copper Square Beam.

3.2 Create a graph for Task 3, similar to the pre-made graph in Task 1.

3.3 Observe the numbers and graph and note any irregularities. Verify that the incremental deflection ($\Delta X$) is relatively constant (linear).
**Task 4. Measure the characteristics of the Unknown Rectangular Beam**

4.1 Repeat all of the steps outlined in Task 1 for the Unknown Rectangular Beam.

4.2 Create a graph of Deflection vs. Force for the unknown rectangular beam.

4.3 Observe the numbers and graph and note any irregularities. Verify that the incremental deflection (ΔX) is relatively constant (linear).

4.4 Calculate the slope of the graph and Young's Modulus of the unknown rectangular beam.

**Task 5. Clean-Up Procedure**

5.1 Remove the Cantilever Beam from the Clamp.

5.2 Leave the apparatus and dial indicator on the table.

5.3 Put items 3-6 (below) back into the plastic storage box.

<table>
<thead>
<tr>
<th>Item No.</th>
<th>Item Description</th>
<th>Qty.</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>Beam Bending Lab Apparatus</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>Dial Indicator</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>Allen Wrench (small)</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>Tee-Handle Hex Driver (yellow handle)</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>Cantilever Beams</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>50-gram Weights</td>
<td>10</td>
</tr>
</tbody>
</table>

**Task 6. Check-Out Policy**

After you have finished the lab and the clean-up procedure, have your instructor or GTA sign the "End-of-Lab Signoff" line at the end of the rubric. You will lose 5 points if this is not signed by your Instructor/TA.
Report Guidelines
ENGR 1181  |  Lab Memo

General Guidelines

Write a Lab Memo
For details on content and formatting, see the Technical Communications Guide on Lab Memo specifications.

Lab Specific Directions

Results & Discussion
- Plot Deflection vs. Applied Force for all four beams on the same graph and discuss the results.
- Calculate the Young’s Modulus of the unknown rectangular beam and identify the material.
- Identify the impact of the various materials and shape configurations.

Conclusion & Recommendations (Individual)
- Each individual engineer is requested to provide their perspective in this area.
- Provide the client with specific recommendations, per their request.